LOCAL HEATING OF A MATERIAL IN THE VICINITY OF A PORE UPON ITS COLLAPSE

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The localization of deformation in the vicinity of nonuniformities upon the high-rate loading of a porous material can result in the onset of local regions for which large velocity gradients, viscous effects, and appreciable heatings are characteristic. The nonequilibrium nature of the heat liberation upon shock compression of porous materials was pointed out in [1-3]. Experimental confirmation of the existence of appreciable temperature gradients in the vicinity of pores in the case of viscoplastic flow of the material in the course of shock-wave loading can be found in [4-6]. A significant dependence of the nonequilibrium temperature of the contacts of particles of a porous material on the size of its fraction with identical loading conditions and initial density has been shown in [7-10]. A significant effect of the particle size in connection with shock-wave initiation of pressed trinitrotoluene on the conditions of formation of sources of local heatings has been discovered in [11, 12].

A theoretical approach to the description of the process of variation of porosity upon the dynamic loading of a medium has been suggested in [13, 14] on the basis of an investigation of the dynamics of deformation of a spherical cell, the ratio between whose inner and outer radii characterizes the porosity of the material. Viscoplastic heating of the material of a spherical cell in the course of the collapse of a pore has been discussed in [15-17]. The variation of the temperature has been obtained in [15] from an analysis of the variation of the specific internal energy of the material on the assumption that the thermal relaxation time appreciably exceeds the collapse time of the pore. Temperature profiles have been constructed in [16, 17] as a result of the solution of the time-dependent thermal balance equation; however, the analysis is limited to small values of the Reynolds number (in the region corresponding to smooth compression of the pore to its equilibrium radius). An analysis of the nature of energy storage and the effects of melting in the vicinity of nonuniformities has been performed in [18] by proceeding from the nature of the internal energy distribution.

The laws of the collapse of a pore in a viscoplastic material and the dynamics of the heating of material in the vicinity of a pore in the course of its collapse are investigated in this paper. The effect of the back pressure of the gas on the nature of deformation of the latter is shown. Relationships are derived which determine the limiting values of the maximum possible heating of the material due to viscoplastic dissipative processes.

We shall consider the process of collapse of a spherical pore acted on by a constant external pressure p. Let a and b be the instantaneous values of the radii of the pore and the spherical cell. The cell material is assumed to be uniform, isotropic, incompressible, and to satisfy the relationships of a viscoplastic medium.

The equations of continuity and motion for the case of central symmetry are written in the form

$$\frac{\partial}{\partial r}(r^2 v) = 0; \tag{1}$$

$$\rho(\partial v/\partial t + v\partial v/\partial r) = \partial\sigma_r/\partial r + (2/r)(\sigma_r - \sigma_\theta), \tag{2}$$

and the determining equation of a viscoplastic material is of the form [17, 18]

$$\sigma_r - \sigma_\theta = \sigma_s + 2\eta(\partial v/\partial r - v/r). \tag{3}$$

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Heating of the material of the spherical cell upon the collapse of the pore is written as a differential thermal conductivity equation in the presence of volume sources of heat liberation

$$c\rho\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial r}\right)=\frac{\lambda}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)-2\sigma_s\frac{v}{r}+12\eta\left(\frac{v}{r}\right)^2, \tag{4}$$

where t is the time; r, an Eulerian coordinate; v, radial component of the velocity vector; σ_r and $\sigma_\theta = \sigma_\varphi$, components of the stress tensor; T, temperature; ρ, λ, and c, density, thermal conductivity coefficient, and specific heat of the material; σ_s, dynamic yield stress; and η, effective viscosity coefficient.

The initial and boundary conditions are of the form

$$\begin{aligned}&\text{at}\quad t=0\ r=r_0,\ T(r)=T_0,\ v(r)=0,\\&\text{at}\quad r=b\ \sigma_r=-p,\ \partial T/\partial r=0,\\&\text{at}\quad r=a\ \sigma_r=-p_q,\ \partial T/\partial r=0,\end{aligned} \tag{5}$$

and the law of pressure variation of the gas adiabatically compressed in the pore is determined in the form

$$p_q=p_{q0}(a_0/a)^{3\gamma}, \tag{6}$$

where p_{q0} is the initial pressure of the gas in the pore and γ is the adiabatic index.

The last two terms on the right-hand side of Eq. (4) reflect the power of heat liberation due to plastic and viscous energy dissipation, respectively. Equating the temperature gradient on the surface of the pore to zero results in a certain understating of the temperature, since the effect of heat exchange of the gas compressed in the pore on heating of the surface layers of the material has not been taken into account. The dynamics of heating of the material upon collapse of the pore will be determined in this case by the competition between the processes of heat liberation due to viscoplastic energy dissipation and heat transfer due to thermal conductivity.

The integral of Eq. (1) is of the form

$$v=\dot{a}(a/r)^2; \tag{7}$$

the dot denotes differentiation with respect to time.

Substituting the last expression and the derivatives $\partial v/\partial t$ and $\partial v/\partial r$ into Eq. (2) and also using (3), (5)-(7), and the incompressibility condition $r^3 - a^3 = r^3_0 - a^3_0$ (the subscript 0 refers to the initial values of the quantities), one can determine the law of variation of the pore radius.

Let us introduce the dimensionless variables and parameters:

$$\begin{aligned}&\tau=(t/a_0)(p/\rho)^{1/2},\ x=a/a_0,\ w_+=\dot{a}(p/\rho)^{-1/2},\ \xi=r/a_0,\\&m_0=(a_0/b_0)^3,\ k=1/m_0-1,\ z=a/b=(1+kx^{-3})^{-1/3},\\&\beta=\sigma_s/p,\ \mathrm{Re}=a_0(p\rho)^{1/2}/\eta,\ q=p_{q0}/p,\ \mathrm{Pr}=c\eta/\lambda,\\&\Theta=c\rho(T-T_0)/p.\end{aligned}$$

Here Re is the Reynolds parameter, Pr is the Prandtl parameter, and m_0 is a parameter which characterizes the initial porosity of the material (the ratio of the specific volume of the pore space to the specific volume of the solid material). The parameter β characterizes the plastic properties of the material, the Reynolds number Re, the viscous properties, and the parameter q, the relationship between the applied pressure and the initial pressure of the gas in the pore.

Changing over in (4) to the new variables and taking (7) into account, we obtain

$$\frac{\partial\Theta}{\partial\tau}+w_+\left(\frac{x}{\xi}\right)^2\frac{\partial\Theta}{\partial\xi}=\frac{1}{\mathrm{Pr\,Re}}\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\Theta}{\partial\xi}\right)-2\beta\frac{w_+x^2}{\xi^3}+\frac{12w_+^2}{\mathrm{Re}}\frac{x^4}{\xi^6}, \tag{8}$$

and the variation law of the pore radius x and its velocity w_+ are determined from the solution of the following system of differential equations:

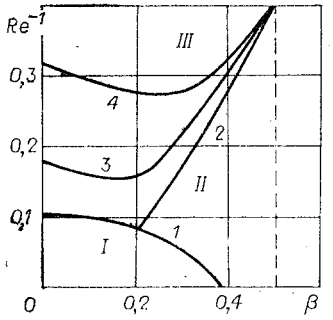

Fig. 1

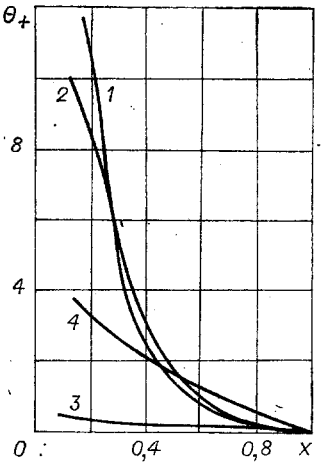

Fig. 2

$$\frac{dw_+}{d\tau}=\left[\frac{(1+z)(1+z^2)}{2}-2\right]\frac{w_+^2}{x}-\frac{4(1+z+z^2)}{\mathrm{Re}\,x^2}w_+-\frac{2\beta\ln z-qx^{-3\gamma}+1}{x(1-z)},\quad \frac{dx}{d\tau}=w_+. \tag{9}$$

The initial and boundary conditions of the problem are:

$$\begin{aligned}&\text{at}\quad \tau=0\ \ x=1,\ w_+=0,\ \Theta(\xi)=0,\\ &\text{at}\quad \xi=x\ \ \partial\Theta/\partial\xi=0,\\ &\text{at}\quad \xi=x/z\ \ \partial\Theta/\partial\xi=0.\end{aligned} \tag{10}$$

The system of equations (8) and (9) with the initial and boundary conditions (10) permits determining the form of the function $\Theta(\tau, \xi)$ for specific values of the parameters m_0, β, Re, q, and Pr.

The characteristics of the collapse of a pore in a viscoplastic material in the absence of gas back pressure have been discussed in [19]. It has been shown that the relationship of the parameters m_0, β, and Re can lead to collapse of the pore (region I in Fig. 1) and its smooth compression to an equilibrium radius (region III in Fig. 1) or to oscillatory motion of the material (region II in Fig. 1). In the latter case the radius of the pore in the compression phase is finite, but less than its equilibrium value. In this case it is necessary for the description of the subsequent motion of the medium to take account of the elastoplastic properties of the material. Curves 1 and 2 in Fig. 1, which separate the regions with different laws of motion of the pore boundary, are determined as a result of the numerical integration of the system of equations (9) for q = 0 and $m_0 = 0.05$. Curves 3 and 4 are obtained with gas back pressure present in the pore for $q = 10^{-4}$ and 10^{-2} ($\gamma = 1.4$) and indicate a narrowing of the region of asymptotic compression of the pore to the equilibrium radius as the parameter q increases; the dashed line bounds the region of values of β corresponding to a state of total plasticity of the cell material:

$$\beta \leqslant \beta_m \equiv -3(1-q)/(2\ln m_0).$$

In the limiting case with $\mathrm{Re} \ll 1$ and neglecting the inertial terms in the first equation of the system (9), one can determine the explicit form of the dependence $w_+(x)$:

$$w_+=\frac{\mathrm{Re}\,x(k+x^3)}{4k}\left[\frac{2}{3}\beta\ln(1+kx^{-3})+qx^{-3\gamma}-1\right]. \tag{11}$$

The law of motion of the pore boundary is determined from (11) in quadratures by the expression

$$\tau=\int_1^x\frac{dx'}{w_+(x')}.$$

With q = 0 the expression for τ is reduced to the form

$$\tau = \frac{2}{\beta \mathrm{Re}} \ln \left[\frac{1 + (2/3)\beta \ln m_0}{1 - (2/3)\beta \ln (1 + kx^{-3})} \right]. \tag{12}$$

Let us determine the value of the maximum possible heating Θ_+, neglecting heat-transfer processes ($\lambda = 0$), which corresponds to the condition $\mathrm{Pr} \to \infty$. Taking the boundary condition (10) at $\xi = x$ into account, and switching from the variable τ to the variable x [in view of the fact that the function $x(\tau)$ is monotonic in the pore compression phase], we obtain from (8) an integral equation for Θ_+

$$\Theta_+ = -2\beta \ln x + \frac{12}{\mathrm{Re}} \int_1^x \frac{w_+(x')}{x'^2} dx'. \tag{13}$$

The first term on the right-hand side of Eq. (13) characterizes the heating associated with plastic deformations of the material, and the second one characterizes the viscous dissipative heating.

As $\mathrm{Re} \to \infty$ (a rigid plastic medium), the maximum dimensionless temperature $\Theta_+ = -2\beta \ln x$. As $x \to 0$, the function $\Theta_+(x)$ has a logarithmic singularity. The indeterminacy of the temperature in the vicinity of the pore produced by plastic deformations can be eliminated if one takes into account that at the melting temperature changes the material into the liquid state with $\beta = 0$ ($\sigma_s = 0$).

In the other limiting case $\mathrm{Re} \ll 1$, substituting the relationship (11) into Eq. (13) and integrating, we obtain

$$\Theta_+ = -2\beta \ln x + q\left[\frac{x^{3(1-\gamma)} - 1}{k(1-\gamma)} - \frac{x^{-3\gamma} - 1}{\gamma}\right] + \frac{1 - x^3}{k} - 3 \ln x + \frac{2\beta}{3k} \times \tag{14}$$

$$\times [(k + x^3) \ln (k + x^3) - (k + 1) \ln (k + 1)] - \beta \ln x \left(3 \ln x + \frac{2}{k} x^3\right) + \frac{2\beta}{3} \int_1^x \frac{\ln (k + x'^3)}{x'^3} d(x'^3).$$

The last integral after expansion of the integrand into a series has the solution

$$I(x) = 3 \ln k \ln x - \sum_{l=1}^{n} (-1)^l \frac{(x^{3l} - 1)}{l^3 k^l}.$$

With small values of the initial porosity $k \gg 1$ and $(x^{3l} - 1) \ll -1$ ($l = 1, 2, \ldots, n$), in accordance with which we obtain

$$I(x) = 3 \ln k \ln x - (1 - x^3)/k.$$

With the last relationship taken into account we obtain after substitution into (14) for Θ_+ the expression

$$\Theta_+ = -2\beta \ln x + q\left[\frac{x^{3(1-\gamma)} - 1}{k(1-\gamma)} - \frac{x^{-3\gamma} - 1}{\gamma}\right] + \frac{1 - x^3}{k} - 3 \ln x + \tag{15}$$

$$+ (2\beta/3k)[(k + x^3) \ln (k + x^3) - (k + 1) \ln (k + 1) - (1 - x^3)] + \beta \ln x (2 \ln k - 3 \ln x - 2x^3/k).$$

It follows from Eq. (15) that, in the region of asymptotic compression of the pore to the equilibrium radius the maximum possible temperature Θ_+ attainable due to viscoplastic energy dissipation does not depend on the parameter Re.

To illustrate the results obtained, the dependence of the dimensionless temperature Θ_+ on the coordinate of the pore boundary x obtained from the solution of the system of equations (9) and (13) is given in Fig. 2. The curves correspond to the following parameters: $m_0 = 0.05$, $\beta = 0.1$, $q = 10^{-4}$, and $\gamma = 1.4$. The Reynolds parameter was taken equal to 8, 6, and ∞ (curves 1-3, respectively). Curve 4 is calculated in accordance with (15) and is the limiting curve for small values of the parameter Re (for $\mathrm{Re} \ll 1$). Increasing the Reynolds parameter leads to a change in the nature of the increase of Θ_+; in the initial stages of the pore collapse process the heating is inappreciable, and in the final stage a sharp increase of the dimensionless temperature is observed. At the same time, for small Re (curve 4), the heating is significant already in the initial stages of the process. The results obtained are valid

up to values of the temperatures not exceeding the melting temperature of the material and permit determining the conditions of formation of the molten zones in the vicinity of the pore as a function of the shock-loading parameters of the porous material.

The extent of the effect of heat transfer on the value of Θ_+ is determined by the relationship between the thermal relaxation time $t_\kappa \sim a^2_0/\kappa$ (κ is the thermal diffusivity of the material) and the characteristic collapse time of the pore $t_\eta \sim a_0(\rho/p)^{1/2}$. Equality of the quantities in question leads to the critical condition Pr Re ~ 1, which permits separating the regimes with a significant and an inappreciable effect of heat transfer on the dynamics of heating of the material in the vicinity of the pore upon its collapse. In the region of parameter values Pr Re >> 1, the effect of heat transfer cannot be taken into account, and the dimensionless temperature Θ_+ will be determined by Eq. (13). In the opposite case, with Pr Re << 1, the maximum value of Θ_+ will depend on the relationship between the rate of heat liberation due to dissipative processes and the heat transfer rate due to thermal conductivity.

In the region of small values of the Reynolds number (Re << 1) the compression time of the pore to the equilibrium radius differs in order of magnitude from t_η by a factor determined by the right-hand side of the expression (12) and appreciably exceeding unity. Taking this fact into account, one can show that with Pr Re << 1 the heat transfer rate can exceed the heat liberation rate already in the initial stage of the pore collapse process, due to which the material in its vicinity will be practically unheated.

The critical value of the pore radius, which determines the boundary between regimes with a different effect of heat transfer on the dynamics of heating of the surface layers of the material, has the form $a^*_0 \simeq \kappa(\rho/p)^{1/2}$, and the levels of the loading pressures should exceed the value $p_m = -(2/3)\sigma_S \ln m_0 + pq_0$, which characterizes the condition of the transition of the material to the plastic state. When $a_0 >> a^*_0$, the effect of heat transfer due to thermal conductivity on the heating of material in the vicinity of a pore is unimportant. For the majority of metals $\rho \simeq 10^4$ kg/m^3 and $\kappa = 10^{-5}$-10^{-4} m^2/sec. In the pressure region p = 1-10 GPa, the critical radius is $a^*_0 \simeq$ 10-0.1 μm, and it agrees in order of magnitude with the experimental results [3, 7-10]. As the pressure increases, the critical value of the pore radius decreases, which results in a decrease of the dependence of the temperature of the surface layers of the material on the initial pore radius. Similar regularities have been noted in [8], in which it is shown that the dependence of the nonequilibrium temperature in powders of copper and nickel on the value of the particle fraction is a maximum at p = 3 GPa, and a further increase in the pressure leads to a weakening of it.

Thus the analysis performed indicates the possibility of the onset of significant temperature gradients in the vicinity of a pore upon its collapse which are caused by viscoplastic dissipative processes.

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A DYNAMIC MODEL OF A THERMOELASTIC CONTINUOUS MEDIUM WITH PRESSURE RELAXATION

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The dynamics of the pulsed heating of a metal to submelting temperatures by an electric current was analyzed in [1] on the basis of the Maxwellian model of a nonlinearly elastic medium with relaxation of shear stresses [2]. A number of experimental relationships which seemed anomalous within the framework of simplified models were explained in this case. In [1] it was assumed that bulk deformations take place elastically.

In the present work a model with relaxation of bulk deformations in a liquid (shear stresses are ignored) is formulated which allows one to give a natural interpretation of an electrical explosion as a phenomenon arising when the level of specific energy content of the medium is inexplicably high. In particular, when a copper conductor is heated in an electric circuit providing a rate of temperature rise $dT/dt \approx 1.5 \cdot 10^{10}$ deg/sec, the starting point of the electrical explosion comes at an energy $Q_* \approx 3.2$ kJ/g ($T \approx 6000°K$), whereas under equilibrium conditions ($p = 1013$ hPa) boiling starts at $Q_{boil} \approx 1.35$ kJ/g ($T \approx 2900°K$).

Such a result cannot be explained within the framework of the theory of metastable states (the Zel'dovich–Fol'mer theory of nucleation, in particular; also see [3]) if plausible estimates are used for the work of formation of the critical nucleus and for the value of the preexponent in the universal expression for the flux of nuclei in the region of sizes larger than the critical size.

Allowance for bulk relaxation also seems a necessary expansion of the model with shear relaxation for the region of lower (submelting) temperatures.

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